

## Analyzing the Case of David Crane

Some would consider Mr. Crane's lesson exemplary. Indeed, Mr. Crane did many things well, including allowing students to construct their own way of solving this cognitively challenging task and stressing the importance of students' being able to explain their reasoning. Students were working with partners and publicly sharing their solutions and strategies with their peers; their ideas appeared to be respected. All in all, students in Mr. Crane's class had the opportunity to become the "authors" of their own knowledge of mathematics.

However, a more critical eye might have noted that the string of presentations did not build toward important mathematical ideas. The upshot of the discussion appeared to be "the more ways of solving the problem, the better," but, in fact, Mr. Crane held each student accountable for knowing only one way to solve the problem. In addition, although Mr. Crane observed students as they worked, he did not appear to use this time to assess what students understood about proportional reasoning or to select particular students' work to feature in the whole-class discussion. Furthermore, he gathered no information regarding whether the two pairs of students who had gotten the wrong answer (Darnell and Marcus, and Missy and Kate) were helped by the student presentations of correct strategies. Had they diagnosed the faulty reasoning in their approaches?

In fact, we argue that much of the discussion in Mr. Crane's classroom was show-and-tell, in which students with correct answers each take turns sharing their solution strategies. The teacher did little filtering of the mathematical ideas that each strategy helped to illustrate, nor did he make any attempt to highlight those ideas. In addition, the teacher did not draw connections among different solution methods or tie them to important disciplinary methods or mathematical ideas. Finally, he gave no attention to weighing which strategies might be most useful, efficient, accurate, and so on, in particular circumstances. All were treated as equally good.

In short, providing students with cognitively demanding tasks with which to engage and then conducting show-and-tell discussions cannot be counted on to move an entire class forward mathematically. Indeed, this kind of practice has been criticized for creating classroom environments in which nearly complete control of the mathematical agenda is relinquished to students. Some teachers misperceived the appeal to honor students' thinking and reasoning as a call for a complete moratorium on teachers' shaping of the quality of students' mathematical thinking. As a result of the lack of guidance with respect to what teachers *could* do to encourage rigorous mathematical thinking and reasoning, many teachers were left feeling that they should avoid telling students anything.

A related criticism of inquiry-oriented lessons concerns the fragmented and often incoherent nature of the discuss-and-summarize phases of lessons. In these show-and-tells, as exemplified in David Crane's classroom, one student presentation would follow another with limited teacher (or student) commentary and no assistance with respect to drawing connections among the methods or tying them to widely shared disciplinary methods and concepts. The discussion offered no mathematical or other reason for students to necessarily listen to or try to understand the methods of their classmates. As illustrated in Mr. Crane's comment at the end of the class, students could simply "pick the way they liked best." This type of situation has led to an increasingly recognized dilemma associated with inquiry- and discovery-based approaches to teaching: the challenge of aligning students' developing ideas and methods with the disciplinary ideas that they ultimately are accountable for knowing.

In sum, David Crane did little to encourage accountability to the discipline of mathematics. How could he have more firmly supported student accountability without undermining student authority? The single most important thing that he could have done would be to have set a clear goal for what he wanted students to learn from the lesson. Without a learning objective in mind, the various solutions that were presented, although all correct, were scattered in the “mathematical landscape.” If, however, he had targeted the learning goal of, for example, making sure that all students recognized that the relationship between caterpillars and leaves was multiplicative and not additive, he might have monitored students’ work with this in mind. Whose work illustrated the multiplicative relationship particularly well? Did the students’ work include examples of different ways of illustrating this relationship—examples that could connect with known mathematical strategies (e.g., unit rate, scaling up)? This assessment of student work would have allowed him to be more deliberate about which students he selected to present during the discussion phase. He might even have wanted to have the incorrect, additive solutions displayed so that students could recognize the faulty reasoning that underlie them. With an array of purposefully selected strategies presented, Mr. Crane would then be in a position to steer the discussion toward a more mathematically satisfying conclusion.

## Conclusion

The Case of David Crane illustrates the need for guidance in shaping classroom discussions and maximizing their potential to extend students’ thinking and connect it to important mathematical ideas. The chapters that follow offer this guidance by elaborating a practical framework, based on five doable instructional practices, for orchestrating and managing productive classroom discussions.